$\boxed{\text{Preg 7}}$ Dado el campo $\bar{f}(x,y,z) = (2z, -2x, x)$ averiguar el flujo del mismo a travez de la superficie $\Sigma \subset \pi_0$: $\pi_0 \equiv x+y+2z=4$ si $\pi_0 \cap x^2+y^2 \leq 9$

En coord. cilindricos $x = \rho \cos\varphi$ $y = \rho \operatorname{Sen}\varphi$ $0 < \varphi \leq 2\pi$; $0 \leq \rho \leq 3$

π_0 : $\bar{P}(\rho,\varphi) = (\rho\cos\varphi, \rho\operatorname{Sen}\varphi; 2 - \frac{1}{2}\rho\cos\varphi - \frac{1}{2}\rho\operatorname{Sen}\varphi)$

$$\iint_\Sigma \bar{f}\cdot\breve{n}\, ds_\Sigma \quad \text{con } \breve{n}\, ds_\Sigma = (\bar{P}_\rho \times \bar{P}_\varphi)\, d\rho\, d\varphi$$

$$\begin{array}{cccc} & \breve{i} & \breve{j} & \breve{k} \\ \bar{P}_\rho = (& \cos\varphi, & \operatorname{Sen}\varphi, & -\frac{(\cos\varphi + \operatorname{Sen}\varphi)}{2}) \\ \bar{P}_\varphi = (& -\rho\operatorname{Sen}\varphi, & \rho\cos\varphi; & \frac{\rho}{2}\operatorname{Sen}\varphi - \frac{\rho}{2}\cos\varphi) \end{array}$$

$$\bar{P}_\rho \times \bar{P}_\varphi = \left(\frac{\rho}{2}\operatorname{Sen}^2\varphi - \cancel{\frac{\rho}{2}\operatorname{Sen}\varphi\cos\varphi} + \frac{\rho}{2}\cos^2\varphi + \cancel{\frac{\rho}{2}\operatorname{Sen}\varphi\cos\varphi}\right)\breve{i} +$$
$$+ \left(\cancel{\frac{\rho}{2}\operatorname{Sen}\varphi\cos\varphi} + \frac{\rho}{2}\operatorname{Sen}^2\varphi - \cancel{\frac{\rho}{2}\operatorname{Sen}\varphi\cos\varphi} + \frac{\rho}{2}\cos^2\varphi\right)\breve{j} +$$
$$+ (\rho\cos^2\varphi + \rho\operatorname{Sen}^2\varphi)\, k$$

$$\bar{P}_\rho \times \bar{P}_\varphi = \frac{\rho}{2}, \frac{\rho}{2}, \rho \qquad (\bar{P}_\rho \times \bar{P}_\varphi)\, d\rho\, d\varphi = \begin{pmatrix} \rho/2 \\ \rho/2 \\ \rho \end{pmatrix} d\rho\, d\varphi = \breve{n}\, ds_\Sigma$$

$$\iint_\Sigma \bar{f}\,\breve{n}\, ds_\Sigma = \int_0^{2\pi}\int_0^3 (\underbrace{4 - \rho\cos\varphi - \rho\operatorname{Sen}\varphi}_{2z}, \underbrace{-2\rho\cos\varphi}_{-2x}, \underbrace{\rho\cos\varphi}_{x}) \begin{pmatrix} \rho/2 \\ \rho/2 \\ \rho \end{pmatrix} d\rho\, d\varphi =$$

$$= \int_0^{2\pi}\int_0^3 \left(2\rho - \frac{\rho^2}{2}\cos\varphi - \frac{\rho^2}{2}\operatorname{Sen}\varphi - \cancel{\rho^2\cos\varphi} + \cancel{\rho^2\cos\varphi}\right) d\rho\, d\varphi =$$

$$= \int_0^{2\pi}\int_0^3 \left(2\rho - \frac{\rho^2}{2}\cos\varphi - \frac{\rho^2}{2}\operatorname{Sen}\varphi\right) d\rho\, d\varphi = \int_0^{2\pi} \left[\rho^2 - \frac{\rho^3}{6}\operatorname{Sen}\varphi - \frac{\rho^3}{6}\cos\varphi\right]_0^3 d\varphi =$$

$$= \int_0^{2\pi} 9\left(1 - \frac{\operatorname{Sen}\varphi}{6} - \frac{\cos\varphi}{6}\right) d\varphi = 9\left[\varphi + \frac{\cos\varphi}{6} - \frac{\operatorname{Sen}\varphi}{6}\right]_0^{2\pi} = \boxed{18\pi}$$

$\boxed{\text{Preg 8}}$ $\bar{f} \in C^2(\mathbb{R}^2)$: $\bar{f}(x,y) = (g(x) + 4y - 3y^2, 4x - 6xy)$ $\exists\, \varphi \mapsto \nabla\varphi = \bar{f}$ $\varphi(1,1) = 5$

$g'' - g = -4x$ $\quad \lambda^2 - \lambda = 0$ $|\lambda| = 1$ $\quad g_h(x) = C_1 e^{-x} + C_2 e^{x}$ la sol part $g_p(x) = Ax + B$

$\Rightarrow g_p' = A$ $\quad g_p'' = 0$ $\quad g'' - g = 0 - Ax - B = -4x \Rightarrow A = 4$ $B = 0$

$$g(x) = C_1 e^{-x} + C_2 e^{x} + 4x \quad \begin{cases} g'(0) = -C_1 + C_2 + 4 = 0 \\ g(0) = C_1 + C_2 = 0 \end{cases} \quad \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$g'(x) = -C_1 e^{-x} + C_2 e^{x} + 4$ $\qquad \Rightarrow C_1 = C_2 = 0$

$$g(x) = 4x \rightarrow \bar{f} = (4x + 4y - 3y^2, 4x - 6xy)$$

Como $\nabla\varphi = \bar{f}$ $\quad \frac{\partial\varphi}{\partial y} = 4x - 6xy \rightarrow \varphi = 4xy - 3xy^2 + C(x)$ (a)

$$\frac{\partial\varphi}{\partial x} = 4y - 3y^2 + C'(x) = 4x + 4y - 3y^2 \rightarrow C'(x) = 4x \rightarrow C(x) = 2x^2 + K$$

Finalmente en (a) $\bar{f}(x,y) = (4x + 4 - 3y^2, 4x - 6xy) = \nabla\varphi(x,y)$

con $\varphi(x,y) = 4xy - 3xy^2 + 2x^2 + K \rightarrow \varphi(1,1) = 4 - 3 + 2 + K = 5 \Rightarrow K = 2$

$$\boxed{\varphi(x,y) = 4xy - 3xy^2 + 2x^2 + 2}$$

Preg 9 $\bar f(x,y) = \left(\frac{1}{y}, -\frac{x}{y^2}\right)$ $D = \{(x,y)\in\mathbb{R}^2 / y<0$

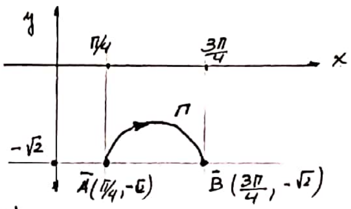
$\Gamma : [x ; -\text{Cosec } x]$ $\pi/4 \le x \le 3\pi/4$ $\bar f \in C^1$ en D

además $\bar f = (f_1(x,y), f_2(x,y))$ se verifica $\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$ $\left(-\frac{1}{y^2} = \frac{d(-x/y^2)}{dx}\right)$

Por Teorema de Schwarte Existe $\varphi(x,y) \in C^2 : \nabla\varphi = \bar f$

$$\frac{\partial\varphi}{\partial x} = \frac{1}{y} \rightarrow \varphi = \frac{x}{y} + C_{(y)} \Rightarrow \frac{\partial\varphi}{\partial y} = -\frac{x}{y^2} + C'_y = -\frac{x}{y^2} \Rightarrow C'_{(y)} = 0 \rightarrow C_y = k$$

$$\boxed{\varphi(x,y) = \frac{x}{y} + k}$$



$$\int_C \bar f\cdot\breve t\, dl = \varphi(\bar B) - \varphi(\bar A) =$$

$$= \left(\frac{3/4\,\pi}{-\sqrt2} + k\right) - \left(\frac{\pi/4}{-\sqrt2} + k\right) = \frac{1}{\sqrt2}\left(\frac{\pi}{4} - \frac{3\pi}{4}\right) = -\frac{2\pi}{4\sqrt2}$$

$$\boxed{\int_\Gamma \bar f\cdot\breve t\, dl = -\frac{\sqrt2\,\pi}{4}}$$

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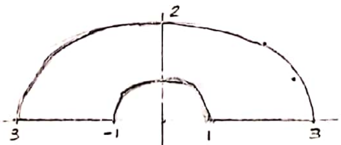
Preg 10 $\bar f \in C^1(\mathbb{R}^2)$ $\bar f(x,y) = (y\,g(xy) + 2y, x\,g(xy) + 4x)$



Notar que $g(xy)$ NO $g(x,y)$ g es función de un producto

$\bar f(x,y) = [f_1(x,y), f_2(x,y)]$ $\begin{cases} \frac{\partial g(x.y)}{\partial x} = y.g' \\ \frac{\partial g}{\partial y} = x\,g' \end{cases}$ $\Rightarrow \frac{\partial g/\partial x}{\partial g/\partial y} = \frac{y}{x}$ $\Rightarrow x\frac{\partial g}{\partial x} = y\frac{\partial g}{\partial y}$

Aplicamos Green:

$$\oint_\Gamma \bar f\cdot\breve t\, dl = \iint_S \left(\frac{\partial f_2(x,y)}{\partial x} - \frac{\partial f_1(x,y)}{\partial y}\right) ds =$$

$$= \iint_S \left((g(x,y) + x\,g'(xy) + 4) - (g(x,y) + y\,g'(x,y) + 2)\right) ds = \iint_S 2\, ds = 2\iint ds = 2.S$$

$$S = \frac{1}{2}\Big(\underbrace{3\times 2.\pi}_{\text{sup elipse}} - \underbrace{\pi 1^2}_{\text{sup cir.}}\Big) = \frac{1}{2}5\pi \qquad 2S = 5\pi$$

$$\boxed{\oint_\Gamma \bar f\cdot\breve t\, dl = 5\pi}$$

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Final 03/08/21

Preg 5 $H \subset \mathbb{R}^3$: $z \ge 1 + x^2 + y^2$ $x \ge 0$ $z \le 5$ usamos coord. cilíndricas:

$dv = \rho\, dz\, d\rho\, d\varphi$ $x = \rho\cos\varphi$ $y = \rho\,\text{Sen}\varphi$ $z = 1 + \rho^2$

$$V = \int_{-\pi/2}^{\pi/2}\int_0^2\int_{1+\rho^2}^5 \rho\, dz\, d\rho\, d\varphi = \int_{-\pi/2}^{\pi/2}\int_0^2 [\rho.z]_{1+\rho^2}^5\, d\rho\, d\varphi = \int_{-\pi/2}^{\pi/2}\int_0^2 5\rho - [\rho + \rho^3]\, d\rho\, d\varphi =$$

$$= \int_{-\pi/2}^{\pi/2}\int_0^2 4\rho - \rho^3\, d\rho\, d\varphi = \int_{-\pi/2}^{\pi/2} \left[2\rho^2 - \frac{\rho^4}{4}\right]_0^2 d\varphi = 4\int_{-\pi/2}^{\pi/2} d\varphi = \boxed{4\pi}$$

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Preg 6 familia uniparamétrica $xy + Cx = 3$ buscamos la familia de ortogonales pasante por el punto $(2,1)$

a) derivamos m.a.m $xy + Cx = 3$ por $x \Rightarrow y + xy' + C = 0$ $C = -y - xy'$

b) eliminamos C entre ambas $\Rightarrow C = \frac{3 - xy}{x} = -y - xy' \Rightarrow 3 - \cancel{xy} = -\cancel{xy} - x^2y'$

nos queda $3 = -x^2y'$

Como la familia $g(x)$ que buscamos debe ser ortogonal a $y(x)$: $xy + Cx = 3$ Condición de ortogonalidad $y' = -\frac{1}{g'}$

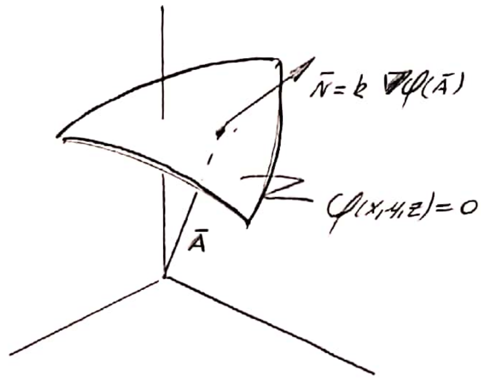
$$\Rightarrow 3 = -x^2\left(-\frac{1}{g'}\right) \rightarrow 3 = \frac{x^2}{dg/dx} \rightarrow dg(x) = \frac{1}{3}x^2dx \quad g(x) = \frac{1}{9}x^3 + K$$

pero pasa por $2,1 \Rightarrow g(2) = \frac{8}{9} + K = 1 \Rightarrow K = \frac{1}{9} \Rightarrow g(x) = \frac{1}{9}x^3 + \frac{1}{9}$

$$g(x) \xrightarrow[x=t]{} \bar X(t) = \left(t ; \frac{t^3 + 1}{9}\right)$$

Preg 11 Sea $\bar{r}_0$ la recta Tangente en $\bar{A}=(0,-1,1)$ a la curva $\bar{x}=\left(\frac{t^3}{4}-2,\ \frac{4}{t}-3,\ \cos(t-2)\right)$ con $t\in\mathbb{R}$ Π_0 plano Tangente a $\varphi(x,y,z)=(x^3+y^3+z^3-xyz=0$ en $\bar{A}$

El vector gradiente a una curva de nivel es normal a la superficie en el punto de la misma donde es evaluado:



$\nabla\varphi=(3x^2-yz,\ 3y^2-xz,\ 3z^2-xy)$

$\nabla\varphi(\bar{A})=(1,3,3)$ y orienta a Π_0

$\bar{A}=(0,-1,1)$

La ecuación Vectorial $(\bar{X}-\bar{A}).\nabla\varphi(\bar{A})=0$

$(x,\ y+1,\ z-1)(1,3,3)=0$

$x+3y+3z=0 : \Pi_0$

La tangente a $\bar{X}(t)$

$\Rightarrow \bar{X}'(t)=\left(\frac{3}{4}t^2,\ -\frac{4}{t^2},\ -\operatorname{Sen}(t-2)\right),\quad \bar{X}(t)=\left(\underbrace{\frac{t^3}{4}-2}\ ,\ \frac{4}{t}-3,\ \cos(t-2)\right)$

$x=0 \Rightarrow \frac{t^3}{4}=2 \rightarrow t=2$

$\bar{X}'(2)=(3,-1,0) \longrightarrow$ vector que orienta la recta $\Big\}\ \bar{r}(\lambda)=\bar{A}+\lambda(3,-1,0)$

pero $\bar{X}(2)=(0,-1,1)\equiv\bar{A}$ y además:

$\nabla\varphi(\bar{A})\cdot\bar{X}'(2)=(1,3,3).(3,-1,0)=0$ son ortogonales

$\Rightarrow \bar{r}(\lambda)\subset\Pi_0$ $\quad\bar{r}(\lambda)$ es subespacio de Π_0 $(\bar{r}(\lambda)\subset\Pi_0)$

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Preg 12 $z=f(x,y)$ tiene plano tangente $6x+4y-2z=7$ en $(3,3,z_0)$

$\Rightarrow z_0=\frac{6.3+4.3-7}{2}=23/2$

Debemos encontrar la derivada direccional de f en 3,3 $f'_{(3,3),v}$.

Como en $(x_0,y_0)=(3,3)$ hemos visto que la existencia de subespacio lineal Tangente $\Rightarrow$ que $f(x,y)$ es DIFERENCIABLE en (x_0,y_0)

Entonces "si y solo si $f(x,y)$ es Diferenciable $f'_{(x_0,y_0),v}=\nabla f(x_0,y_0)\cdot\breve{v}$

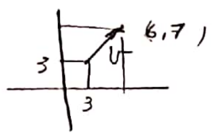
$\varphi(x,y,f(x,y))=0\Rightarrow\nabla\varphi(\bar{P}_0)=k.N$ ← normal al plano.

$N=(6,4,-2)$ orienta al plano

Como $z=f(x,y)$ $\varphi=(-f(x,y)+z)\Rightarrow\nabla\varphi=\left(-\frac{\partial f}{\partial x},-\frac{\partial f}{\partial y},1\right)$

$\left(-\frac{\partial f}{\partial x},-\frac{\partial f}{\partial y},1\right)=K(6,4,-2)\Rightarrow 1=K(-2)\Rightarrow K=-1/2$

$-\frac{\partial f}{\partial x}=-3$ y $\frac{\partial f}{\partial y}=-2 \Rightarrow \boxed{\nabla f_{(3,3)}=(3,2)}$



Como $\bar{v}=(6,7)-(3,3)=3,4$ $\quad\breve{v}=\frac{\bar{v}}{\|\bar{v}\|}=\frac{3}{5},\frac{4}{5}$

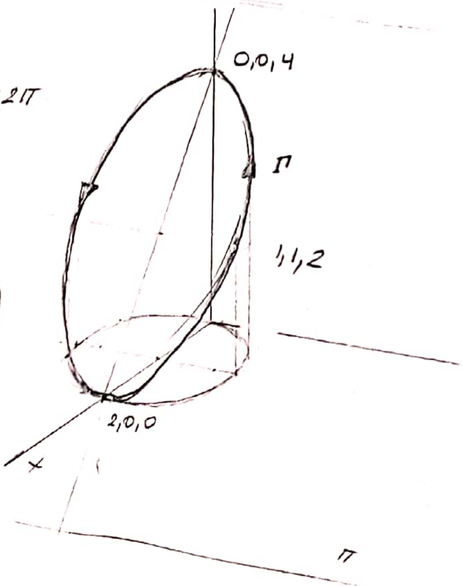
$\frac{Df(3,3)}{D\bar{v}}=f'_{(x_0,y_0),v}=\nabla f_{(3,3)}\cdot\breve{v}=(3,2).\left(\frac{3}{5},\frac{4}{5}\right)=\frac{9}{5}+\frac{8}{5}=\boxed{\frac{17}{5}}$

Preg 4 $\bar f: \mathbb{R}^3 \to \mathbb{R}^3 \,/\, \bar f(x,y,z) = (-x^2y\,,\, xy^2\,;\, x^2\, Sen(z^8))$ $\quad \Gamma: x^2+y^2+z=4 \wedge (x-1)^2+y^2=1$

como $x^2+y^2=2x \Rightarrow \Gamma$ es una elipse y $\Gamma \subset \Pi$ plano $2x+z-4=0$

donde $\bar N = 2,0,1 \Rightarrow \breve n = \left(\frac{2\sqrt5}{5}, 0, \frac{\sqrt5}{5}\right)$ y se puede aplicar Stokes

Si Parametrizamos la sup. encerrada por Γ en plano π $\quad z=4-2x$



$$\bar P(u,v) = (1+u\cdot Cos\,v\,;\, u\cdot Sen\,v\,;\, 2-2u\, Cos\,v)$$

$0 \le u \le 1,\ 0 \le v < 2\pi$

$$\begin{array}{lccc} & \breve i & \breve j & \breve k \\ \bar P_u = (& Cos\,v\,, & Sen\,v\,, & -2\,Cos\,v \\ \bar P_v\ (& -u\,Sen\,v, & u\,Cos\,v & 2u\,Sen\,v \end{array}$$

$$\bar P_u \times \bar P_v = \left(2u(Sen^2 v + Cos^2 v)\,;\, \overbrace{2u\,Cos\,v\,L\,v - 2u\,Cos\,v\,L\,v}^{0}\,;\, u(Cos^2 v + L^2 v)\right)$$

$$\breve n\, ds = \bar P_u \times \bar P_v\, du\, dv = (2u,\, 0,\, u)\, du\, dv$$

$$\bar\nabla \times \bar f = \begin{vmatrix} \breve i & \breve j & \breve k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -x^2y & x\,y^2 & x^2\,Sen(z^8) \end{vmatrix}$$

$$\bar\nabla \times \bar f = (0,\, -2x\,Sen(z^8)\,;\, y^2+x^2)$$

$(1+u\,Cos\,v)^2 + (u\,Sen\,v)^2 = 1+2u\,Cos\,v + u^2$

Stokes: $$\oint_\Gamma \bar f\cdot \breve t\, dl = \int_S (\bar\nabla\times\bar f)\,\breve n\, ds = \int_0^{2\pi}\!\!\int_0^1 (0,\, -2x\,Sen(z^8),\, x^2+y^2)\begin{pmatrix} 2u \\ 0 \\ u \end{pmatrix} du\, dv$$

$$= \int_0^{2\pi}\!\!\int_0^1 \underbrace{u(1+2u\,Cos\,v + u^2)}\, du\, dv = \int_0^{2\pi}\!\!\int_0^1 (u + 2u^2\,Cos\,v + u^3)\, du\, dv =$$

$$= \int_0^{2\pi}\left[\frac{u^2}{2} + \frac{u^4}{4} + \frac{2}{3}u^3\,Cos\,v\right]_0^1 dv = \int_0^{2\pi}\left(\frac34 + \frac23\,Cos\,v\right)dv = \left[\frac34 v + \frac23\,Sen\,v\right]_0^{2\pi}$$

$\frac34 \cdot 2\pi$

$$\boxed{\oint_\Gamma \bar f\cdot\breve t\, dl = \frac{3\pi}{2}}$$

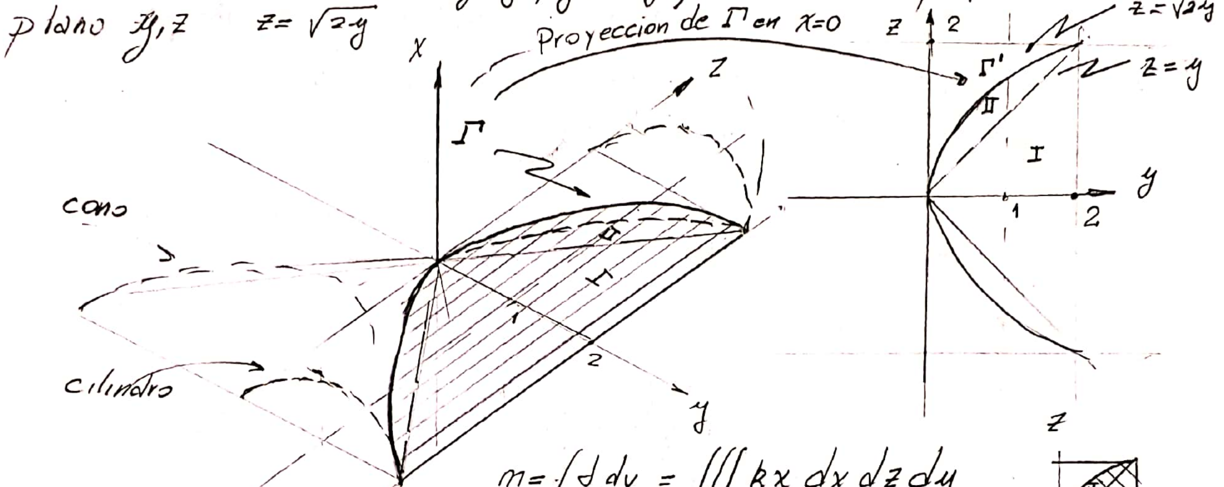
Problema 3 como $m=\int_V d\,dv$ Tenemos que resolver una integral de volumen sobre D $z^2=x^2+y^2 \cap x^2+y^2=2y \Rightarrow x^2+(y-1)^2=1 \cap x\geq 0 \Rightarrow d=|x|k=kx$

- El cono y el cilindro nos invitan a usar coordenadas cilíndricas pero el cilindro no tiene el mismo eje que el cono y nos metemos en un lío

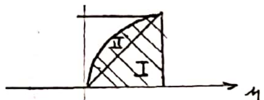
Adoptamos el plano $x=0$ como base para la integración

Cono y Cilindro intersecan generando $\Gamma: z^2=x^2+y^2 \cap x^2+y^2=2y \Rightarrow z=\sqrt{2y}$

z es bivaluada $\Gamma=(\sqrt{2y-y^2},\, y,\, \sqrt{2y})$ Esta curva proyecta sobre plano y,z $\quad z=\sqrt{2y}$



$$m=\int_V d\,dv=\iiint kx\,dx\,dz\,dy$$



Por la simetría de la Figura integraremos en

en zona I vamos de $x=0$ a $x=\sqrt{2y-y^2}$

en zona II vamos desde sup lateral del Cono a sup lat cilindro.

$x=\sqrt{z^2-y^2}$ a $x=\sqrt{2y-y^2}$ muy fácil!

$$m=2\cdot\Bigg[\underbrace{\int_0^2\int_0^y\int_0^{\sqrt{2y-y^2}} kx\,dx\,dz\,dy}_{\text{Integro sobre zona I}}+\underbrace{\int_0^2\int_y^{\sqrt{2y}}\int_{\sqrt{z^2-y^2}}^{\sqrt{2y-y^2}} kx\,dx\,dz\,dy}_{\text{integro sobre zona II}}\Bigg]$$

$$m=\cancel{2}k\int_0^2\left[\int_0^y\left[\frac{x^2}{\cancel{2}}\right]_0^{\sqrt{2y-y^2}}dz+\int_y^{\sqrt{2y}}\left.\frac{x^2}{\cancel{2}}\right]_{\sqrt{z^2-y^2}}^{\sqrt{2y-y^2}}dz\right]dy=$$

Los radicales se van!

$$m=k\int_0^2\left[\int_0^y(2y-y^2)\,dz+\int_y^{\sqrt{2y}}2y-\cancel{y^2}-(z^2-\cancel{y^2})\,dz\right]dy=$$

$$=k\int_0^2\left[\left[(2y-y^2)z\right]_0^y+\left.2yz\right]_y^{\sqrt{2y}}-\left.\frac{z^3}{3}\right]_y^{\sqrt{2y}}\right]dy=$$

$$=k\int_0^2\left(\cancel{2y^2}-y^3+2y\sqrt{2y}-\cancel{2y^2}-\frac{2y\sqrt{2y}}{3}+\frac{y^3}{3}\right)dy=$$

cuentita

$$=k\int_0^2\frac{2^{5/2}}{3}y^{3/2}-\frac{2}{3}y^3\,dy=k\left[\frac{2^{5/2}}{3}\frac{y^{5/2}}{5/2}-\frac{2}{3}\frac{y^4}{4}\right]_0^2=$$

$$m=k\left[\frac{2^{7/2}}{15}\cdot 2^{5/2}-\frac{2}{3}\frac{16}{4}\right]=k\left(\frac{2^6}{15}-\frac{8}{3}\right)=k\left(\frac{64}{15}-\frac{8}{3}\right)=\frac{k\,\cancel{24}^{\,8}}{\cancel{15}_{\,5}}$$

$$\boxed{m=\frac{8}{5}k}$$

bien a lo guapo, sin compu y sin calcu, Como rendía mi querido viejo.